

# CSU FRESNO MATH PROBLEM SOLVING

February 28, 2009

## Part 1: Counting & Probability

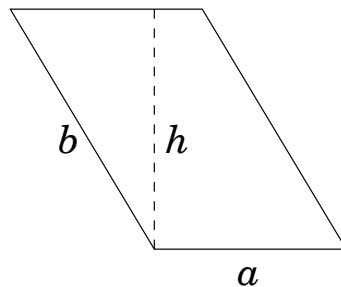
- (MH 11-12 2008) Suppose we draw 100 horizontal lines and 100 vertical lines in the plane. How many “pieces” of the plane are formed by cutting along all of these lines? Note that some of the pieces may have infinite area.
  - 10000
  - 10001
  - 10004
  - 10201
  - 10204
- (MH 11-12 2008) How many subsets of  $\{a, b, c, d, e, f, g\}$  contain both  $a$  and  $b$ ?
  - 32
  - 25
  - 16
  - 12
  - 9
- (simplified MH 11-12 2008) The number of positive integers that are divisors of  $720 = 2^4 \cdot 3^2 \cdot 5$  is
  - 16
  - 24
  - 25
  - 27
  - 30
- (MH 11-12 2008) The number of **even** positive integers that are divisors of  $720 = 2^4 \cdot 3^2 \cdot 5$  is
  - 15
  - 16
  - 24
  - 25
  - 29
- (MH 9-10 2008) Ten people are attending a meeting. If each shakes hands with each other person exactly once, how many handshakes will occur?
  - 20
  - 45
  - 90

- (d) 100
6. (MH 9-10 2008, MH 11-12 2008) At a party, every two people shook hands once. How many people attended the party if there were exactly 66 handshakes?
- (a) 65
  - (b) 54
  - (c) 33
  - (d) 22
  - (e) 12
7. (MH 9-10 2008) Find the second term in the expansion of  $(x + y)^{25}$ .
- (a)  $2300x^{22}y^3$
  - (b)  $300y^{223}x^2$
  - (c)  $300x^{23}y^2$
  - (d)  $25x^{24}y$
8. (MH 11-12 2008) In how many ways can you walk up a stairway with 6 steps if you can take one or two steps at a time?
- (a) 9
  - (b) 10
  - (c) 11
  - (d) 12
  - (e) 13
9. (MH 9-10 2008) The land of Xod has coins that are regular triangular pyramids. The four faces are labeled N, G, H, and S. A Xodian is tossing two coins. What is the probability that both coins land with the same side facing down?
- (a)  $\frac{1}{2}$
  - (b)  $\frac{1}{4}$
  - (c)  $\frac{1}{6}$
  - (d)  $\frac{1}{8}$
10. (MH 11-12 2008) What is the probability of rolling a red die and a blue die and having the number showing in the red die to be larger than the number showing in the blue one?
- (a)  $\frac{4}{9}$
  - (b)  $\frac{1}{2}$
  - (c)  $\frac{19}{36}$
  - (d)  $\frac{2}{3}$
  - (e)  $\frac{5}{12}$
11. (MH 11-12 2008) A pair of dice is thrown. What is the probability that the two numbers that appear differ by exactly 2?
- (a)  $\frac{2}{3}$

- (b)  $\frac{1}{3}$   
(c)  $\frac{1}{6}$   
(d)  $\frac{2}{9}$   
(e)  $\frac{1}{9}$
12. (MH 9-10 2008) A jar contains 3 red marbles, 4 blue marbles, and 5 green marbles. Janice takes one marble out of the jar, and then Tom takes one marble out of the same jar. What is the probability that Janice drew a red marble and Tom drew a green marble?
- (a)  $\frac{5}{44}$   
(b)  $\frac{5}{48}$   
(c)  $\frac{5}{33}$   
(d)  $\frac{5}{36}$
13. (MH 11-12 2008) An urn contains three white and four black balls. We take out a ball and put it in a drawer without looking at it. After that we take out a second ball. Find the probability that this ball is white.
- (a)  $\frac{1}{6}$   
(b)  $\frac{3}{7}$   
(c)  $\frac{5}{6}$   
(d)  $\frac{1}{3}$   
(e)  $\frac{1}{7}$
14. (MH 9-10 2008) A color-blind individual has 16 pairs of socks, 10 identical red pairs and 6 identical navy blue pairs. After washing his socks, he just throws them in the sock drawer without pairing them up. If he randomly selects two socks, what is the probability that they will be the same color?
- (a)  $\frac{95}{248}$   
(b)  $\frac{33}{248}$   
(c)  $\frac{16}{31}$   
(d)  $\frac{1}{2}$
15. (MH 9-10 2008) A population starts with a single amoeba. For this one and for the generations thereafter, there is a probability of  $\frac{3}{4}$  that an individual amoeba will split to create two amoebas, and a  $\frac{1}{4}$  probability that it will die out without producing offsprings. What is the probability that the family tree of the original amoeba will go on for ever?
- (a)  $\frac{1}{2}$   
(b)  $\frac{1}{3}$   
(c)  $\frac{2}{3}$   
(d)  $\frac{3}{4}$

## Part 2: Pythagorean Theorem, Area, and Volume

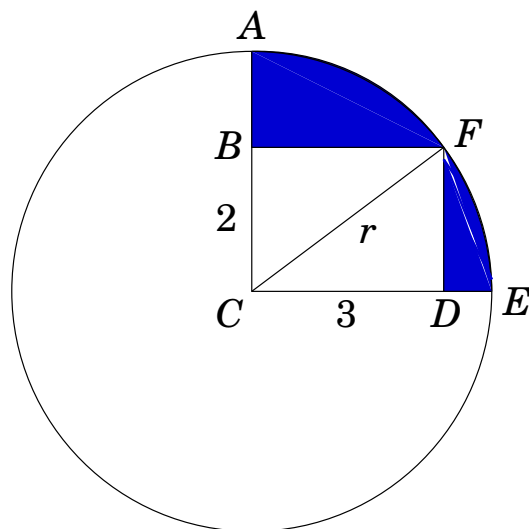
- (MH 9-10 2008) The hypotenuse  $c$  and one side  $b$  of a right triangle are consecutive integers. The square of the other leg of the triangle is
  - $bc$
  - $\frac{c}{b}$
  - $c + b$
  - $c - b$
- (MH 11-12 2008) Two perpendicular lines, intersecting at the center of a circle of radius 1, divide the circle into four parts. A smaller circle is inscribed in one of those parts. What is the radius of the smaller circle?
  - $\frac{1}{3}$
  - $\frac{2}{5}$
  - $\frac{1}{2}$
  - $\sqrt{2} - 1$
  - $2 - \sqrt{2}$
- (MH 11-12 2008) Determine the length of the diagonals of the parallelogram shown if  $a = 6$  in,  $b = 10$  in and  $h = 8$  in.



- 10 in and 8 in
  - 10 in and 6 in
  - $2\sqrt{137}$  in and 8 in
  - $4\sqrt{13}$  in and 8 in
  - $4\sqrt{13}$  in and  $4\sqrt{13}$  in
- (MH 9-10 2008) The ratio of the area of a square inscribed in a semicircle to the area of the square inscribed in the entire circle is
    - 1:2
    - 2:3
    - 2:5
    - 3:4
  - (MH 11-12 2008) Two parallel chords in a circle have lengths 6 and 8. The distance between them is 1. Then the diameter of the circle is

- (a)  $10\sqrt{3}$
- (b) 14
- (c) 12
- (d) 10
- (e) 9

6. (MH 11-12 2008)  $C$  is the center of the circle and  $F$  is a point on the circle such that  $BCDF$  is a 2 in by 3 in rectangle. What is the area of the shaded region? (in square inches).



- (a)  $\frac{13\pi}{2} - 5$
- (b)  $\frac{13\pi}{4} - 5$
- (c)  $\frac{13\pi}{2} - 6$
- (d)  $\frac{13\pi}{4} - 6$

7. (MH 9-10 2008) A circle of radius 6 is inscribed in a regular hexagon. If the area of the hexagon is  $x\sqrt{3}$ , what is  $x$ ?

- (a) 75
- (b) 72
- (c) 71
- (d) 70

8. (MH 11-12 2008) Which of the following shapes has the largest area?

- (a) A right triangle with legs of length 6 and 8, and hypotenuse of length 10
- (b) A square with side of length 5
- (c) A circle with radius of length 3
- (d) A rectangle with sides of length 3 and 9

9. (MH 9-10 2008) If the length of each side of a triangle is increased by 20%, then the area is increased by what percent?

- (a) 40%

- (b) 44%
- (c) 48%
- (d) 52%
10. (MH 11-12 2008) When the base of a triangle is decreased 10% and the altitude is increased 10%, then the area is
- (a) Unchanged
- (b) increased 10 %
- (c) decreased 10 %
- (d) increased 1 %
- (e) decreased 1 %
11. (MH 11-12 2008) If the width of a particular rectangle is doubled and the length is increased by three, then the area is tripled. What is the length of the rectangle?
- (a) 1
- (b) 2
- (c) 3
- (d) 6
- (e) 9
12. (MH 11-12 2008) A square has perimeter  $p$  and area  $A$ . If  $A = 2p$ , then what is the value of  $p$ ?
- (a) 54
- (b) 48
- (c) 36
- (d) 32
- (e) 24
13. (MH 11-12 2008) The volume of a large (solid) cube is 125 cubic inches. A new shape is formed by removing a 1 in  $\times$  1 in  $\times$  1 in cube from one corner of the large cube. The surface area of this new shape in square inches is
- (a) 250
- (b) 225
- (c) 180
- (d) 150
- (e) 120
14. (MH 9-10 2008) A metal tank in the shape of a right circular cylinder is one-fourth full of water. If 80 mL of water is added, it will be one-third full. What is the volume of the tank?
- (a) 960 mL
- (b) 320 mL
- (c) 240 mL
- (d) It can't be determined from the information given.

15. (MH 9-10 2008) A company sells peanut butter in cylindrical jars. Marketing research suggests that using wider jars will increase sales. If the diameter of the jars is increased by 25% without altering the volume, by what percent must the height be decreased?
- (a) 10
  - (b) 25
  - (c) 36
  - (d) 50

See solutions at <http://zimmer.csufresno.edu/~mnogin/mfd-prep.html>