MATH 75B

Test 2

March 27, 2019

Name:_____

- No books, notes, or calculators are allowed.
- Please show all your work for problems 7-10.

Multiple choice questions: circle the correct answer

1. Evaluate $\lim_{x \to 0} \dfrac{\sin(6x)}{3x}$.

   A. $\frac{1}{2}$ **B.** 2 **C.** 3 **D.** 9
   E. none of the above

2. Find the linear approximation to $f(x) = x^2$ at $a = 3$.

   A. $L(x) = 2x$ **B.** $L(x) = 2x - 6$ **C.** $L(x) = 3x + 6$ **D.** $L(x) = 6x - 9$
   E. none of the above

3. Find the largest possible area of a rectangle with perimeter 20.

   A. 5 **B.** 16 **C.** 24 **D.** 25
   E. none of the above

4. The Mean Value Theorem can be applied to the function $f(x) = \dfrac{x}{(x-1)(x-3)}$ on which of the following intervals?

   A. $[-2, 0]$ **B.** $[0, 2]$ **C.** $f(x) = (1, 3)$ **D.** $f(x) = [2, 4]$
   E. none of the above

5. Which of the following functions is concave downward everywhere in its domain?

   A. $f(x) = x^2$ **B.** $f(x) = e^x$ **C.** $f(x) = \ln(x)$ **D.** $f(x) = \sin(x)$
   E. none of the above

6. Which of the following is the formula for finding x_{n+1} given x_n according to Newton's method?

   A. $x_n + \dfrac{f(x_n)}{f'(x_n)}$ **B.** $x_n + \dfrac{f'(x_n)}{f(x_n)}$ **C.** $x_n - \dfrac{f(x_n)}{f'(x_n)}$ **D.** $x_n - \dfrac{f'(x_n)}{f(x_n)}$
   E. none of the above

Regular problems: show all your work

7. A rectangular box with a square base and open top (i.e., no top, just the bottom and four sides) must have the volume of 500 in^3. Find the dimensions of the box that minimize the surface area.

8. Use L'Hospital's rule to evaluate the following limits:

(a) $\lim_{x \to 0} \dfrac{e^x - x - 1}{3x^2}$

(b) $\lim_{x \to \frac{\pi}{2}^-} (\tan x - \sec x)$

9. Suppose you want to use Newton's method to approximate the value of the root of $e^x = 4 - x$.

   (a) What would be a good choice of x_0? Justify your answer.

   (b) If you choose $x_0 = 0$, then what x_1 do you get?

10. Let $f(x) = \dfrac{(x-2)^2}{x-1}$. Find the following:

(a) domain

(b) intercepts

(c) vertical and horizontal asymptotes

(d) critical numbers

(e) intervals of increase and decrease

(f) local maximum and minimum values

(g) sketch the graph of $f(x)$: