

November 15, 2017

Probability

1. An urn contains 3 white and 5 black balls. One ball is drawn. What is the probability that it is black?
2. From a deck of cards, four cards are drawn at random. What is the probability that they will be the four face cards (J,Q,K,A) of the same suit?
3. Two dice are thrown. What is the probability that the total score shown is a seven?
4. A coin is tossed n times. What is the probability that heads will appear an odd number of times?
5. Six boys and six girls sit in a row randomly. Find the probability that
 - (a) the six girls sit together.
 - (b) the boys and girls alternate.
6. Five persons entered the elevator on the first floor of a 9-floor building. Suppose that each of them independently and with equal probability, can leave the elevator at any floor beginning with the second floor. Find the probability that all five people leave at different floors.
7. A man and a woman appear in an interview for two vacancies in the same position. The probability of the man's selection is $1/4$ and that of the woman's selection is $1/3$. What is the probability that
 - (a) both of them will be selected.
 - (b) only one of them will be selected.
 - (c) none of them will be selected.
8. If four whole numbers are selected at random and multiplied together, show that the chance that the units digit in the product is 1, 3, 7, or 9 is $16/625$.
9. What is the chance that a leap year selected at random will have 53 Sundays?
10. A and B are two independent witnesses (that is, there is no collusion between them) in a case. The probability that A will speak the truth is x and the probability that B will speak the truth is y . A and B agree in a certain statement. Show that the probability that the statement is true is
$$\frac{xy}{1 - x - y + 2xy}$$
11. Suppose the probability for A to win a game against B is 0.4. If A has an option of playing either a "best of 3 games" or a "best of 5 games" match against B , which option should A choose so that the probability of his winning the match is higher? (Assume that no game ends in a draw.)

12. A coin is tossed $(m + n)$ times ($m > n$). Show that the probability of at least m consecutive heads is

$$\frac{n + 2}{2^{m+1}}.$$

13. What is the probability that in a group of N people at least two of them will have the same birthday?
14. A set A has n elements. A subset P of A is selected at random. Returning the elements of P the set A is formed again, and then a subset Q of the set A is selected at random. Find the probability that P and Q have no common elements.