Fresno Math Circle **Grades 11-12** **Fall 2017**

December 13, 2017

Probability with continuous variables

1. A stick is broken into two pieces at a random place. What is the probability that the shorter piece is at least $\frac{1}{4}$ of the original length?

2. Suppose that two points are chosen at random in the unit interval $[0, 1]$. What is the probability that both are to the right of $\frac{1}{3}$?

3. Two points x and y are chosen at random in the unit interval $[0, 1]$. What is the probability that $y - x > \frac{1}{2}$?

4. A disk of radius 1 is placed randomly entirely inside a 10×10 square $ABCD$. What is the probability that it is entirely contained in
   (a) the bottom half of the square?
   (b) triangle ABC?

5. In a common carnival game a player tosses a penny from a distance of about 5 feet onto the surface of a table ruled in 1-inch squares. If the penny ($\frac{3}{4}$ inch in diameter) falls entirely inside the square, the player receives 5 cents but does not get his/her penny back; otherwise he/she loses the penny. If the penny lands on the table, what is the probability of winning?

6. Two points are selected at random on a fixed circle. What is the probability that the length of the chord joining them exceeds the radius of the circle?

7. Duels in the town of Discretion are rarely fatal. There, each contestant comes at a random moment between 5 A.M. and 6 A.M. on the appointed day and leaves exactly 5 minutes later, honor served, unless his opponent arrives within the time interval and then they fight. What fraction of duels lead to violence?

8. A stick is broken at random in two places. What is the probability that each of the three obtained pieces is at least $\frac{1}{5}$ of the original length?

9. A stick is broken at random in two places. What is the probability that a triangle can be formed using the three obtained pieces?

10. Numbers b and c are chosen randomly in the unit interval $[0, 1]$. What is the probability that the quadratic equation $x^2 + bx + c = 0$ has real roots?

11. (Problem Solving Contest for College Students, Fresno State, 2017) Real numbers a and b are chosen randomly and independently in the interval $[-1, 1]$. Find the probability that the line $y = ax + b$ and the parabola $y = x^2$ intersect.

12. A table of infinite expanse has inscribed on it a set of parallel lines spaced a units apart. A needle of length l (smaller than a) is twirled and tossed on the table. What is the probability that when it comes to rest it crosses a line?