

Sample Test 3

1. A graph $K_{k,l,m}$ has $k+l+m$ vertices divided into three sets: k vertices in one set, l vertices in another set, and m vertices in the third set. Two vertices are connected if and only if they are in different sets. Prove that $K_{1,3,5}$ has a Hamilton path but not a Hamilton cycle.
2. The fifth-graders are going to visit Kindergarteners and read books to them. Each fifth-grader will be reading a book to one Kindergartner. There are 20 children in each class. How many ways are there to pair up each fifth-grader with a Kindergartner?
3. How many seven-digit binary (base 2) numbers have no more than three 1s in them?
4. Find an equation of the line with a negative slope and passing through the point $(1, 1)$ such that the triangle bounded by this line and the axes is divided by the parabola $y = x^2$ into two regions of equal area.

- **Extra credit:** Show that for any real numbers a_1, \dots, a_{145} ,

$$a_1 \cos x + a_2 \cos(2x) + \dots + a_{145} \cos(145x)$$

cannot take on only positive values.