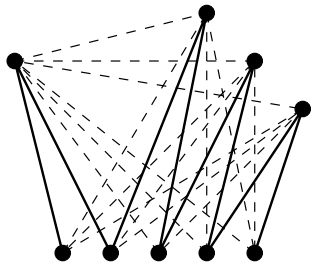
Math 145

Practice Test 3 - Solutions

1. See problem 9 in chapter 10.
2. *A Hamilton path is shown:*



There is no Hamilton cycle because among 9 vertices, 5 are in one set. Thereofe if a Hamilton cycle existed then at least 2 of these 5 would be consecutive in the cycle. However, they cannot be joined because since they are in one set.

3. See problem 15 in chapter 14.
4. (Suggestion: draw a picture.) Let the line have slope m, then the equation of the line is $y = mx - m + 1$. The x-intercept (found by setting $y = 0$ and solving for x) is $x = \dfrac{m-1}{m}$, and the y-intercept (found by setting $x = 0$) is $y = -m + 1$. Therefore the area of the triangle is

$$A_1 = \frac{1}{2} \cdot \frac{m-1}{m} \cdot (-m+1) = -\frac{(m-1)^2}{2m}.$$

The area of the region bounded by the y-axis, the parabola, and the line is

$$\begin{aligned} A_2 &= \int_0^1 (mx - m + 1 - x^2)\, dx \\ &= \frac{mx^2}{2} - mx + x - \frac{x^3}{3}\Big|_0^1 \\ &= \frac{m}{2} - m + 1 - \frac{1}{3} \\ &= -\frac{m}{2} + \frac{2}{3} \\ &= -\frac{3m-4}{6}. \end{aligned}$$

Since we need $2A_2 = A_1$, we have

$$\begin{aligned} -\frac{3m-4}{3} &= -\frac{(m-1)^2}{2m} \\ 2m(3m-4) &= 3(m-1)^2 \\ 3m^2 - 2m - 3 &= 0 \\ m &= \frac{1 \pm \sqrt{10}}{3}. \end{aligned}$$

Since we need the negative solution, we have $m = \dfrac{1-\sqrt{10}}{3}$.

- Hint: see problem 21 in chapter 10.