

MATH 111

Practice Test 1

Note: the actual test will consist of five or six questions (some with two or three parts).

- Review all terms, notations, and types of proofs studied in chapters 0–3.
- Let $U = \{x \in \mathbb{Z} \mid 0 \leq x \leq 10\}$ be the universal set, $A = \{x \in U \mid x \text{ is even}\}$, $B = \{1, 2, 3, 4, 5\}$.
 - Draw a Venn diagram that illustrates the above sets.
 - Determine (i.e. list all the elements of) the following sets: $A \cap B$, \overline{A} , $A \cup \overline{B}$.
 - How many elements does $A \times B$ have?
 - List any three elements of $A \times B$.
- Let $A = \{1\}$, $B = \{2\}$, $C = \{\{3\}\}$, $D = \{1, \{2\}, \{1, 2, 3\}\}$.
 - Which of the following statements are true: $A \in D$, $A \subset D$, $B \in D$, $B \subset D$, $C \in D$, $C \subset D$, $\emptyset \in D$, $\emptyset \subset D$?
 - What are the cardinalities of these four sets?
- Let $A_n = \left[\frac{1}{n}, \frac{n+1}{n} \right)$ for each $n \in \mathbb{N}$. Determine $\cup_{n \in \mathbb{N}} A_n$ and $\cap_{n \in \mathbb{N}} A_n$ (no formal proof is required, but please provide an explanation of your answer; a picture might be helpful).
- Determine the truth values of the following statements (where $x, y, z \in \mathbb{R}$).
 - $\exists x (x^2 = 8)$
 - $\forall x (x \neq -x)$
 - $\exists x (x^2 - x + 1 = 0)$
 - $\forall x (x^2 + 1 > 0)$
 - $\forall x \forall y (xy = 0)$
 - $\exists x \exists y (xy = 0)$
 - $\forall x, y (x \neq y)$
 - $\forall x \forall y \forall z (z = x + y)$
 - $\exists x, y, z (z = x + y \wedge x \neq y)$
 - $\forall x, y (x \leq y \vee x \geq y)$

6. (a) Show that $P \Leftrightarrow Q$ and $(P \wedge Q) \vee ((\neg P) \wedge (\neg Q))$ are logically equivalent.
- (b) The compound statement $(P \Leftrightarrow Q) \Leftrightarrow ((P \wedge Q) \vee ((\neg P) \wedge (\neg Q)))$ is a _____.
- (c) The compound statement $(P \Leftrightarrow Q) \Leftrightarrow \neg((P \wedge Q) \vee ((\neg P) \wedge (\neg Q)))$ is a _____.
7. Let n and m be integers. Prove the following statements and state what types of proof you used.
- (a) If $3n^2 + 5n$ is odd, then $n \geq 10$.
- (b) If n is even, then $3n^2 - 2n - 5$ is odd.
- (c) If $n - 5m$ is odd, then n and m are of the opposite parity.
8. Let x be a real number. Prove the following statements and state what types of proof you used.
- (a) If $x > -7$, then $-5 - x^2 < 0$.
- (b) If $|x| = 5$, then $x^2 + x + 1 > 20$.