MATH 111

Sample Final

The sample exam given below is similar in length and difficulty to the actual final exam. After you review all topics covered in this class, find two hours when you will not be disturbed, and try to do this sample exam. Review more as needed. Take your time to finish this sample exam.

Note: since this is only a sample final, it does not cover all topics you have to review. Make sure you review all of the topics listed on the next page.

1. Prove or disprove the following statement:
   Let A, B, and C be sets. Then $(A \cup B) - C = (A - C) \cup (B - C)$.
2. Determine whether the compound propositions $(P \vee Q) \Rightarrow (P \wedge Q)$ and $P \Leftrightarrow Q$ are logically equivalent.
3. Let $n \in \mathbb{Z}$. Prove that if $3n^2 + 4n + 2$ is even, then n is even.
4. Prove or disprove the following statement:
   For any $a \in \mathbb{Z}$, the number $a^3 + a + 100$ is positive.
5. Consider the relation R defined on $\mathbb{Z}$ by aRb iff $ab \leq 0$. Determine whether R is
   (a) reflexive,
   (b) symmetric,
   (c) transitive,
   (d) an equivalence relation.
6. Consider the function $f : \mathbb{Z} \to \mathbb{Z}$ defined by $f(x) = \begin{cases} x & \text{if } x \text{ is even,} \\ 2x & \text{if } x \text{ is odd.} \end{cases}$
   Determine whether f is
   (a) injective,
   (b) surjective,
   (c) bijective.
7. Prove that the number 111 cannot be written as the sum of four integers, two of which are even and two of which are odd.
8. The following problem could be a problem for extra credit on the final exam.
   Give an example of a bijective function $f : \mathbb{Z} \to \mathbb{N}$ and find its inverse.

Final Exam Study Guide

- Communicating mathematics properly (chapter 0)
- Sets, including sets of numbers and their notations, subsets, set operations (union, intersection, difference, complement, product), and their notations and fundamental properties, Venn diagram, partitions (chapter 1)
- Propositions, propositional functions, logical operations (and, or, not, implication, biconditional) and their notations, truth tables, tautologies, contradictions, logical equivalence and its fundamental properties, quantifiers (universal and existential), including multiple quantifiers (chapter 2)
- Types of proofs: trivial, vacuous, by cases, direct, by contrapositive, by contradiction (chapters 3–5)
- Definitions and properties of divisibility and congruences (sections 4.1–4.2)
- Various proof techniques for statements involving: sets, integers, rational/irrational numbers, positive/negative numbers, any real numbers, absolute value (chapters 4–5)
- Proof of irrationality of $\sqrt{2}$ (section 5.2)
- Mathematical Induction—classical and "more general" forms (sections 6.1–6.2); the strong induction is good to know for your later classes, but will not be needed on this exam.
- Testing, proving and disproving (quantified) statements (including: when an example/counterexample is sufficient? When a general proof is required?) (chapter 8)
- Relations and their properties (reflexive, symmetric, transitive), equivalence relations, equivalence classes (chapter 9)
- Functions and their properties (injective, surjective, bijective), composition of functions, inverse functions (chapter 10)