Math 111 Test 1 October 4, 2021

Reminder: your camera must be on for the duration of the test.

1. (8 pts) Consider $A = [0, \infty)$ and $B = (-1, 2)$ to be subsets of the universal set $\mathbb{R}$. Determine the following sets. Give your answers in the interval notation.

   (a) $A \cup B$

   (b) $\overline{A} \cap B$

   (c) $A - B$

   (d) $(A \cap \overline{A}) \cup B$

   No justification is needed for this problem.

2. (4 pts) Let $S = \{a, b\}$ and $T = \{a, b, c\}$. List all elements of $S \times T$.

   No justification is needed for this problem.

3. (6 pts) Two or more of the following compound statements are logically equivalent. Which ones? List all that apply.

   i. $P \Rightarrow Q$

   ii. $(\neg P) \Rightarrow (\neg Q)$

   iii. $(\neg Q) \Rightarrow (\neg P)$

   iv. $(\neg P) \vee Q$

   v. $(\neg P) \wedge Q$

   No justification is needed for this problem.

4. (12 pts) Let $P(x) =$ "x is even" and $Q(x) =$ "$x \geq 5$" where $x \in \mathbb{Z}$. Determine the truth values of the following statements. Provide brief justifications.

   (a) $P(1) \Rightarrow Q(1)$

   (b) $\forall x \in \mathbb{Z}\ (Q(x) \Rightarrow Q(x+1))$

   (c) $\exists x \in \mathbb{Z}\ (P(x) \Leftrightarrow Q(x))$

5. (8 pts) Let $x \in \mathbb{R}$. Prove that if $x^2 - 2x + 2 < 0$, then $0 < x < 1$. Write a complete proof.

6. (12 pts) Let $n \in \mathbb{Z}$. Prove that $3n+8$ is even if and only if n is even. Write a complete proof.

- (For extra credit, 8 pts) Give an example of a compound statement that has the following truth table.

P	Q	R	Compound statement
T	T	T	F
T	T	F	T
T	F	T	T
T	F	F	F
F	T	T	T
F	T	F	F
F	F	T	F
F	F	F	F