MATH 111

Test 3

December 11, 2007

Name:

- No books, notes, or calculators are allowed.

- Please show all your work.

1. (6 points) Let A and B be nonempty sets. Is it true or false that every function from A to B is also a relation from A to B? Explain.

2. (10 points) Let A be a set and $f : A \to A$ be one-to-one. Prove that $f \circ f$ is one-to-one.

3. (12 points) Determine whether $f : \mathbb{N} \to \mathbb{N}$ defined by $f(x) = x^2 + 3$ is one-to-one; onto; bijective.

4. (12 points) Let R be a relation on $\mathbb{R}$ defined by $(a, b) \in R$ if and only if $a + b \in \mathbb{Z}$. Determine whether R is reflexive; symmetric; transitive; an equivalence relation. If it is an equivalence relation, describe its distinct equivalence classes.

5. (10 points) Recall that the factorial of n is defined as $n! = 1 \cdot 2 \cdot 3 \cdot \ldots n$. Prove that for any positive integer n,

$$1 \cdot 1! + 2 \cdot 2! + 3 \cdot 3! + \ldots n \cdot n! = (n+1)! - 1.$$

6. (**For extra credit**, 8 points) Give an example of a bijective function from $\mathbb{Q}$ to $\mathbb{Q} - \{0\}$.