MATH 111

Test 3

December 10, 2007

Name:

- No books, notes, or calculators are allowed.

- Please show all your work.

1. (4 points) Let $A = \{1, 2, 3\}$ and $B = \{4, 5, 6\}$. Give an example of a relation from A to B that is not a function from A to B.

2. (12 points) Let A be a set and $f : A \rightarrow A$ be onto. Prove that $f \circ f$ is onto.

3. (12 points) Determine whether $f : \mathbb{R} \to \mathbb{R}$ defined by $f(x) = \dfrac{1}{x^2 + 1}$ is one-to-one; onto; bijective.

4. (12 points) Let R be a relation on $\mathbb{Z}$ defined by $(a, b) \in R$ if and only if $2|(a + b)$. Determine whether R is an equivalence relation. If so, describe its distinct equivalence classes.

5. (10 points) Prove that for any positive integer n,

$$1 \cdot 2 \cdot 3 + 2 \cdot 3 \cdot 4 + \ldots + n(n+1)(n+2) = \frac{n(n+1)(n+2)(n+3)}{4}.$$

6. (**For extra credit**, 8 points) Let A be a set. Prove that if a function $f : A \to A$ is an equivalence relation on A, then it is bijective.