MATH 110 **Homework 6**

1. Let $U = \{a, b, c, d, e, f, g\}$, and let f be an interpretation of formulas with P, Q, and R in U defined by $f(P) = \{a, b, c\}$, $f(Q) = \{c, d, e\}$, $f(R) = \{b, d, f\}$. Find the following.
   (a) $f(\neg R)$
   (b) $f(P \wedge Q)$
   (c) $f(P \vee Q \vee R)$
   (d) $f(P \to R)$
2. For the above interpretation ($U = \{a, b, c, d, e, f, g\}$, $f(P) = \{a, b, c\}$, $f(Q) = \{c, d, e\}$, $f(R) = \{b, d, f\}$):
   (a) Find a compound statement that is mapped to the whole set U but is not a tautology.
   (b) Also find a compound statement that is mappted to the empty subset but is not a contradiction.
3. Let $U = \{1\}$ (containing just one element).
   (a) The compound statement $P \to Q$ is not a tautology, therefore there exists an interpretation that sends $P \to Q$ to a proper subset of U, i.e. the empty set. Find an interpretation that sends $P \to Q$ to the empty subset.
   (b) On the other hand, the compound statement $P \to Q$ is not a contradiction, therefore there exists an interpretation that sends $P \to Q$ to a non-empty subset of U, i.e. the whole set U. Find an interpretation that sends $P \to Q$ to U.
4. Recall that $\mathbb{N}$ denotes the set of natural numbers (positive integers) and $\mathbb{R}$ denotes the set of all real numbers. Also let $S = \{-1, 1\}$. Determine the truth value of the following statements. Provide a brief justification.
   (a) $\forall x \in S\ x^2 = 1$
   (b) $\forall x \in \mathbb{R}\ x^2 = 1$
   (c) $\forall x \in \mathbb{N}\ x^2 > 0$
   (d) $\forall x \in \mathbb{R}\ x^2 > 0$
5. Recall also that $\mathbb{Z}$ denotes the set of integer numbers. Determine the truth value of the following statements.
   (a) $\exists x \in S\ x^2 = 3$
   (b) $\exists x \in \mathbb{N}\ x^2 = 3$
   (c) $\exists x \in \mathbb{Z}\ x^2 = 3$
   (d) $\exists x \in \mathbb{R}\ x^2 = 3$