# Fundamental Identities in Set Theory

Let $U$ be the universal set and $A$, $B$, and $C$ be any subsets of $U$. The following identities can be verified (illustrated) using Venn diagrams, and formally proved by showing that each side is a subset of the other.

1. Commutative laws:
$$A \cup B = B \cup A$$
$$A \cap B = B \cap A$$

2. Associative laws:
$$(A \cup B) \cup C = A \cup (B \cup C)$$
$$(A \cap B) \cap C = A \cap (B \cap C)$$

3. Distributive laws:
$$A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$$
$$A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$$

4. Idempotent laws:
$$A \cup A = A$$
$$A \cap A = A$$

5. Identity laws:
$$A \cup \emptyset = A$$
$$A \cap U = A$$

6. Inverse laws:
$$A \cup \overline{A} = U$$
$$A \cap \overline{A} = \emptyset$$

7. Domination laws:
$$A \cup U = U$$
$$A \cap \emptyset = \emptyset$$

8. Absorption laws:
$$A \cup (A \cap B) = A$$
$$A \cap (A \cup B) = A$$

9. Double complement law:

$$\overline{(\overline{A})} = A$$

10. DeMorgan's laws:

$$\overline{A \cup B} = \overline{A} \cap \overline{B}$$

$$\overline{A \cap B} = \overline{A} \cup \overline{B}$$

11. Difference identity:

$$A - B = A \cap \overline{B}$$

12. "Contrapositive" identity:

$$A - B = \overline{B} - \overline{A}$$

13. Symmetric difference identities:

$$(A - B) \cup (B - A) = (A \cup B) \cap (\overline{A} \cup \overline{B})$$

$$(A - B) \cup (B - A) = (A \cup B) - (A \cap B)$$