

1. Let $U = \{a, b, c, d, e, f, g\}$, and let f be an interpretation of formulas with P , Q , and R in U defined by $f(P) = \{a, b, c\}$, $f(Q) = \{c, d, e\}$, $f(R) = \{b, d, f\}$. Find the following.
 - (a) $f(\neg R)$
 - (b) $f(P \wedge Q)$
 - (c) $f(P \vee Q \vee R)$
 - (d) $f(P \rightarrow R)$
2. For the above interpretation ($U = \{a, b, c, d, e, f, g\}$, $f(P) = \{a, b, c\}$, $f(Q) = \{c, d, e\}$, $f(R) = \{b, d, f\}$):
 - (a) Find a compound statement that is mapped to the whole set U but is not a tautology.
 - (b) Also find a compound statement that is mapped to the empty subset but is not a contradiction.
3. Let $U = \{1\}$ (containing just one element).
 - (a) The compound statement $P \rightarrow Q$ is not a tautology, therefore there exists an interpretation that sends $P \rightarrow Q$ to a proper subset of U , i.e. the empty set. Find an interpretation that sends $P \rightarrow Q$ to the empty subset.
 - (b) On the other hand, the compound statement $P \rightarrow Q$ is not a contradiction, therefore there exists an interpretation that sends $P \rightarrow Q$ to a non-empty subset of U , i.e. the whole set U . Find an interpretation that sends $P \rightarrow Q$ to U .
4. Determine the truth value of the following statements.
 - (a) $\exists x \in \mathbb{R} x^2 = 0$
 - (b) $\forall x \in \mathbb{R} x^2 = 0$
 - (c) $\forall x \in \mathbb{R} x^2 > 0$
 - (d) $\forall x \in \mathbb{N} x^2 > 0$
5. Determine the truth value of the following statements.
 - (a) $\exists x \exists y \in \mathbb{Z} x^2 = y$
 - (b) $\forall x \forall y \in \mathbb{Z} x^2 = y$
 - (c) $\exists x \exists y \in \mathbb{N} x^2 + y^2 = 3$
 - (d) $\exists x \exists y \in \mathbb{R} x^2 + y^2 = 3$