The Power of Three Trick

What the audience sees.

You leave the room and three volunteers come up to the board and write any three numbers from 1 to 9. Then they multiply each number by 3 and write the resulting products below these three numbers. They take each digit in the products and multiply by 3 again and write the resulting products below them. One more audience member comes up to the board and crosses out one of the digits from the final list of numbers. The audience adds up the remaining digits on the last line and the volunteers then erase the board. You are asked to come in to the room and are told this sum. You tell everyone the value of the digit that the audience member crossed out.

How you do it.

You ask volunteers from the audience to come up to the board and write any three numbers from 1 to 9 once you have exited the room. You ask them to multiply each of the three numbers by 3 and record the corresponding products on the board. You also ask them to multiply each of the digits in the resulting products again by 3 and record their resulting products. One more audience member is then asked to cross out one of the digits and the audience adds up the rest of the digits. It is important that the audience memorizes the crossed out digit. At this time, you come back into the room and are told the sum that the audience just computed. You find the difference between that sum and the next multiple of 9. Announce this difference as the crossed out digit. Indeed, that is the correct digit!

Why it works.

Suppose the three numbers from 1 to 9 that the volunteers wrote in the beginning are a, b, and c. When they multiply the numbers by 3, then the products are $3a, 3b$, and $3c$. Each of these numbers is divisible by 3. By the divisibility test of 3, the sum of the digits of each of $3a$, $3b$, and $3c$ is divisible by 3. So, when each of the digits is multiplied by 3 again, the sum of the numbers formed by multiplying each of the digits by 3 is three times the sum of the digits of $3a, 3b$, and $3c$ and is, therefore, divisible by 9. Since the sum of the digits of a number has the same remainder when divided by 9 as the number itself, it implies that the sum of the digits in the numbers formed by multiplying the digits of $3a, 3b$, and $3c$ by 3 is also a multiple of 9. Note that none of the digits in this final result can be a zero (since we started with three numbers from 1 to 9). If one of the digits is crossed out, say x, then the sum of the remaining digits is x less than the next multiple of 9. In particular, if the sum is already divisible by 9, then x is 9. So, when you are told the sum of the remaining digits, all you have to do is find out the difference between the sum you are told and the next multiple of 9 and it will represent the crossed out digit.

Example.

- Suppose the volunteers wrote the numbers $3, 7$ and 4 on the board.
- They multiply each of these by 3 to get $9, 21$, and 12. Since each of these three numbers is divisible by 3, the sum of their digits $9 + 2 + 1 + 1 + 2 = 15$ is also divisible by 3.

- They multiply each digit (9, 1, 2, 1, and 2) again by 3 to get $27, 6, 3, 3$, and 6. Since the sum $27 + 6 + 3 + 3 + 6 = 3(9 + 2 + 1 + 1 + 2)$, we get that the sum of the these numbers is divisible by 9 as the sum $9 + 2 + 1 + 1 + 2$ was divisible by 3.

- The sum of the digits of each of $27, 6, 3, 3$, and 6 is $9, 6, 3, 3$, and 6 and these sums have the same remainder as $27, 6, 3, 3$, and 6 when divided by 9. Since the sum of $27, 6, 3, 3$, and 6 was divisible by 9, the sum of their digits is also divisible by 9.

- Now suppose one of the audience members crosses out the digit 6 from this last line of numbers. The remaining digital sum is $2 + 7 + 3 + 3 + 6 = 21$. When you are told this sum, you compute the difference between this number and the next multiple of 9 which is $27 - 21 = 6$. You then announce that the crossed out digit is 6.